

Application of the Kalman Filter to PHENIX Charged Particle Tracking

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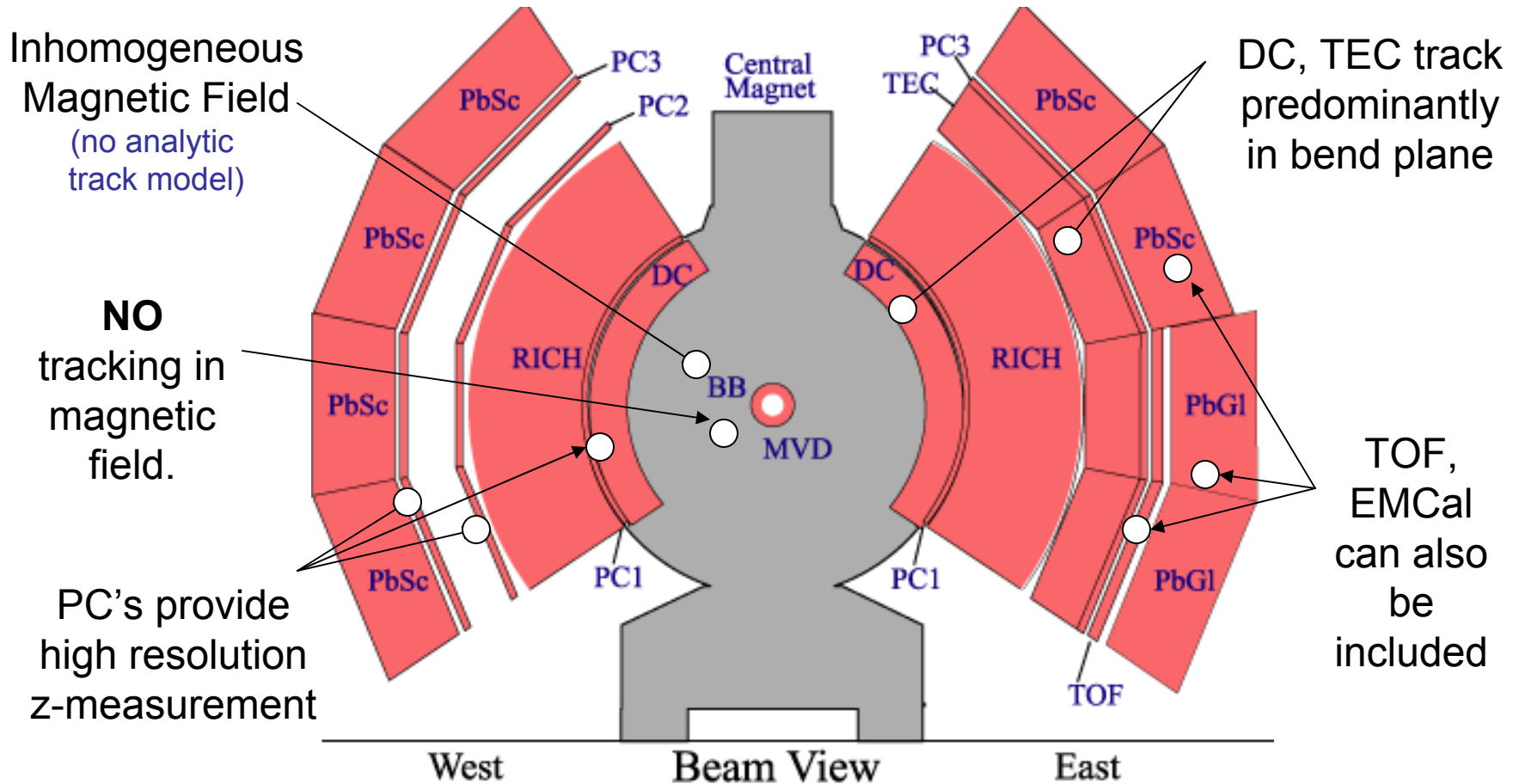
- **What is the Kalman Filter?**
- **Tracking in PHENIX**
 - fit procedure
 - vertex/momentum fit
 - single particle MC results
 - including multiple scattering

References: R.E. Kalman, Transactions of the ASME, 1960
R. Fruhwirth, NIM A262 (1987) 444-450
P. Billoir, Comp. Phys. Comm. 57(1989) 340

What is the Kalman Filter?

- Recursive technique for obtaining the solution to a least-squares fit.
- Avoids large matrix inversion in traditional fits, does not require analytic fit function.
- Adaptable to include both **measurement** and **process noise**.
- Used for tracking ballistic missiles, facial recognition and robotic piloting. The HEP community has largely adopted it as a standard (BaBar, CLEO, ALICE,...)
- Highly adaptable – can be used for tracking, track fitting, and merging information from different detector systems.

Tracking in PHENIX



Momentum reconstruction currently done with only DC and PC1.

Fit Procedure

- Fit track from outside-in (provides best-fit parameters at vertex).
- Approximate PHENIX as a set of measurements made at constant R.
- Seed trajectory is generated from CGL (**C**entral **G**lobal tracking) vertex and momentum, trajectory is described by a parameter vector at each radial position along the track:

$$\eta = (\varphi, z, \varphi_p, \theta_p, p)$$

- The track is transported to location of each hit $\eta \rightarrow \eta'$. This is done numerically for the track parameters and covariance matrix C (linearized approximation, known as “extended Kalman fit”).

$$T = \frac{\partial \eta'}{\partial \eta} = \text{Transport matrix}$$

$$C' = TCT^T$$

Fit Procedure (cont.)

- Kalman gain matrix is calculated from measurements, measurement error, and state vector covariance matrix.
- Track state vector is updated according to the gain.

K = Kalman gain matrix

V = covariance matrix of hit to be added

H = projects state vector onto measurements

m = matrix of measurements

$$K = CH(V + HCH^T)^{-1}$$

$$\eta' = \eta + K(m - H\eta)$$

$$C' = (I - KH)C$$

- Actual measurements directly update the relevant state vector parameters. Derivatives of elements in the state vector are updated through the influence of the covariance and transport matrices).

Vertex and Momentum

- Once we are through the DC, we have the best estimate of the momentum vector of the track, but we know nothing about the magnitude.
- We have no measurements between the vertex and the DC, so we are forced to assume the track came from the vertex.
- Define a chi-square associated with the vertex as a function of the momentum magnitude:

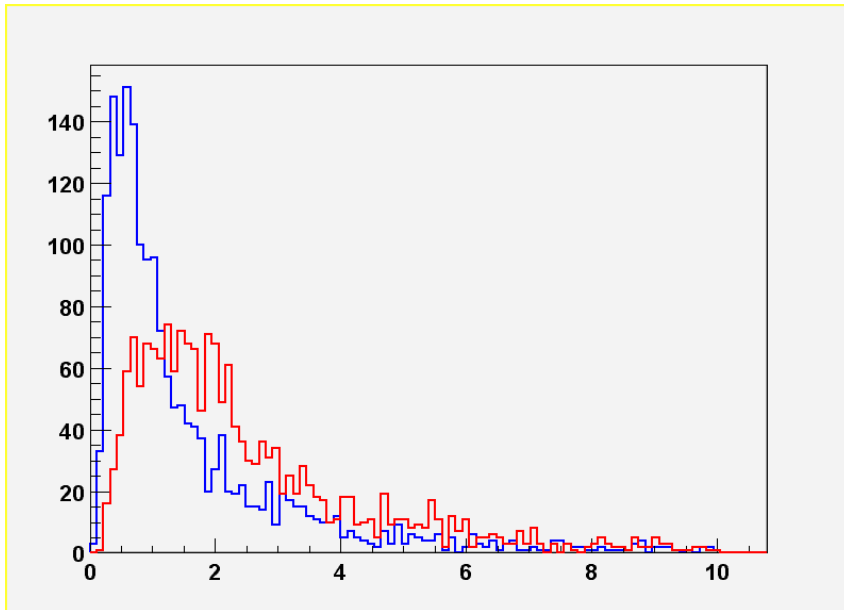
$$\chi_{VTX}^2(p) = \left(\frac{\Delta r}{\sigma_r}\right)^2 + \left(\frac{\Delta z}{\sigma_z}\right)^2$$

Vertex and Momentum (cont.)

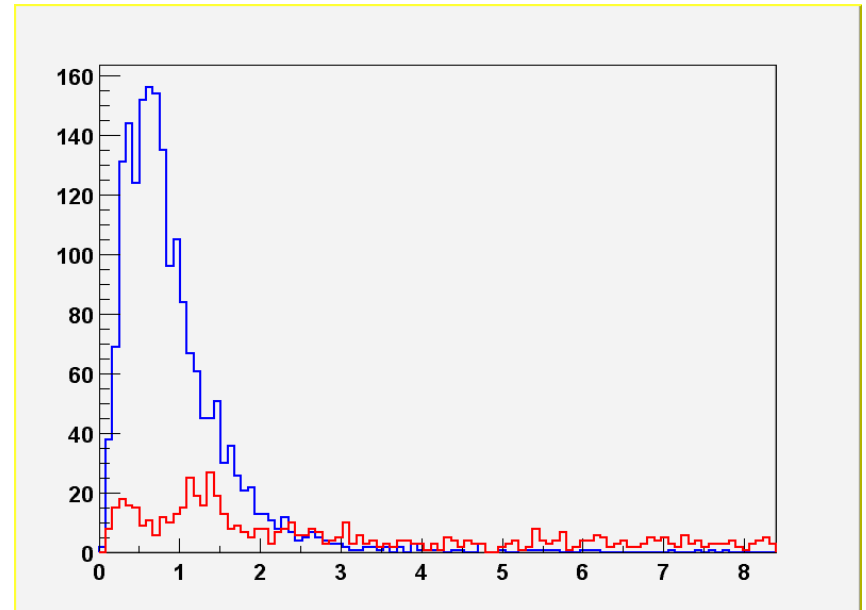
- One-dimensional minimization problem – use the gsl solver to obtain the best-fit momentum.
- Each minimization step involves propagating the particle through the magnetic field to the closest approach to the vertex.
- Note that this procedure fixes the entry vector to the magnetic field, and only varies the total momentum.
- Computationally expensive!

MC χ^2 Distributions

Red: before fit
Blue: after fit



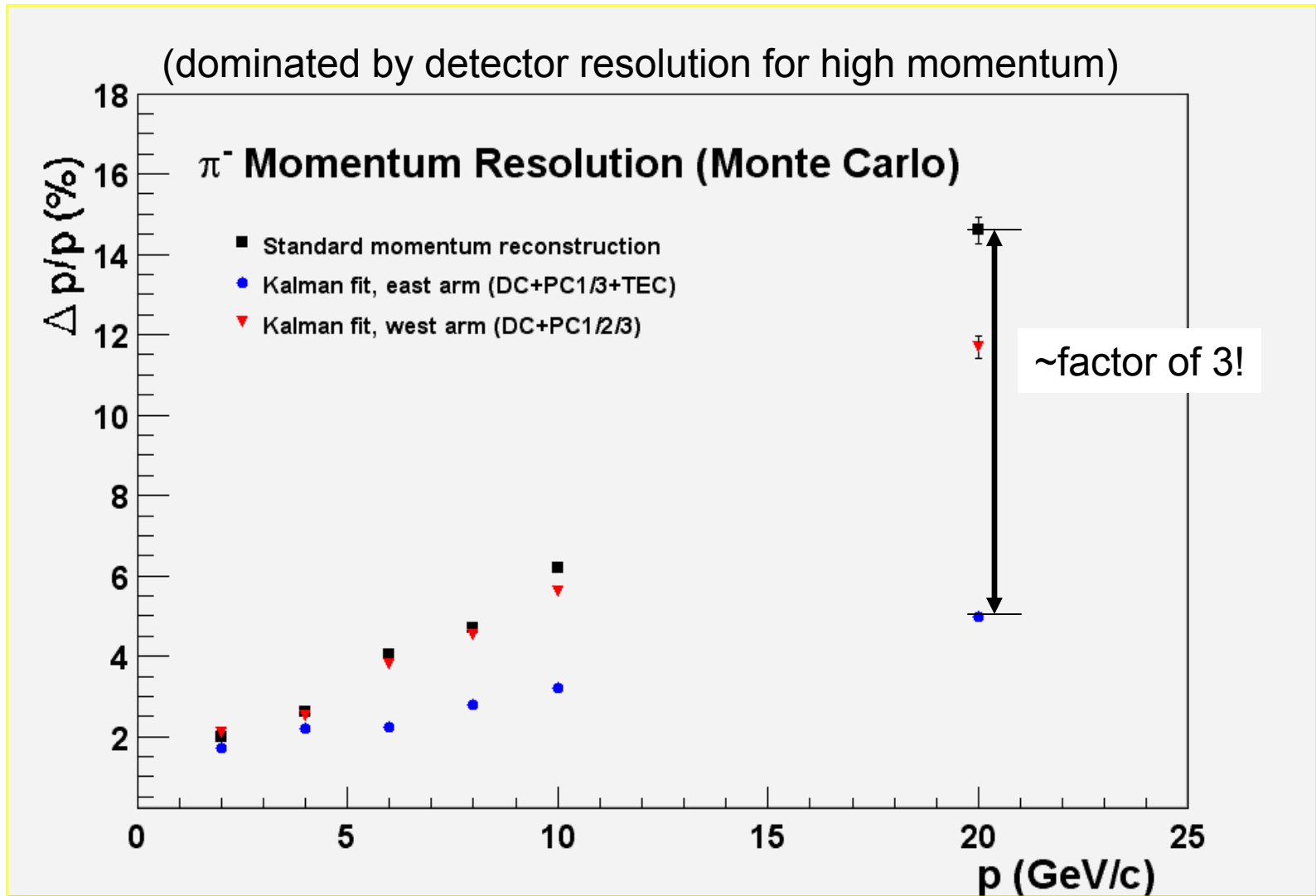
$\chi^2(\phi)$ per d.o.f



$\chi^2(z)$ per d.o.f

- 10 GeV/c π^- , east arm (TEC)
- Track consistency with all detectors improved.

Momentum Resolution ($p_T > 2 \text{ GeV}/c$)



Conclusions

- Significant improvement in momentum resolution at high p_T by including all PHENIX detectors in a Kalman filter.
- Statistically proper chi-square for track selection.
- Extensions for low- p_T possible (multiple scattering and energy loss).

Multiple Scattering

- Multiple scattering introduces a correlation between measurements through the correlation matrix.
- Since we're tracking inwards, we need to **reduce** the covariance matrix as we add subsequent hits. Multiple scattering treated in two planes defined by the momentum vector.

θ_0 = multiple scattering angle

$$V_{\theta_p \theta_p} = \theta_0^2$$

$$V_{\phi_p \phi_p} = \frac{\theta_0^2}{\cos^2 \theta_p}$$

- Continuous media treated as a sequence of multiple steps.

Momentum Resolution ($p_T=0.5$ GeV/c)

(dominated by detector resolution for low momentum)

0.5 GeV/c π^+ , East arm

With MS correction
resolution approaches
that of CGL
reconstruction

$(\delta p/p) \sim 1\%$

